

Economics 100B: Macroeconomics

Monetary Policy II: Drivers of Rate Change and Closed-Form Impulse Response Solution

October 9, 2020

Reading:

- R.J. Hawkins (2020) – *A Monetary Policy Model*.
On bCourses as **MonetaryPolicyModelNotes-v7.pdf**.

The Optimal Rate Rule

Today's Agenda:

- ① The rate rule and changes to:
 - The components of r^* .
 - The components of the inflation gap.
- ② Is this an inflation-targeting rule?

The Optimal Rate Rule

The components of r^ .*

The response of the rate rule

$$r_t = r^* + \frac{1}{\zeta_Y \left(\gamma + \frac{1}{\gamma\beta} \right)} \left(\pi_t - \pi^T \right)$$

where

$$r^* = \frac{1}{\zeta_Y} \left(\bar{Y} - Y^P \right)$$

is a function of $\bar{Y}$:

$$Y = \underbrace{\frac{(\bar{C} + \bar{I} + \bar{G} + \bar{NX} - mpc \times \bar{T} - \bar{S})}{(1 - mpc)}}_{\bar{Y}} - \underbrace{\frac{(\zeta_C + \zeta_I + \zeta_{NX})}{(1 - mpc)}}_{\zeta_Y} r$$

The Optimal Rate Rule

The components of r^ .*

The response of the rate rule

$$r_t = r^* + \frac{1}{\zeta_y \left(\gamma + \frac{1}{\gamma\beta} \right)} \left(\pi_t - \pi^T \right)$$

where

$$r^* = \frac{1}{\zeta_Y} \left(\bar{Y} - Y^P \right)$$

is a function of Y^P :

$$Y^P = AK^\alpha L^{1-\alpha} = K^\alpha (EL)^{1-\alpha}$$

The Optimal Rate Rule

The components of the inflation gap.

The response of the rate rule

$$r_t = r^* + \frac{1}{\zeta_Y \left(\gamma + \frac{1}{\gamma\beta} \right)} \left(\pi_t - \pi^T \right)$$

where

$$r^* = \frac{1}{\zeta_Y} \left(\bar{Y} - Y^P \right)$$

is a function of π_t :

$$\pi_t = \pi_{t-1} + \gamma \left(y_t - y^P \right) + \rho_t$$

The Optimal Rate Rule

The components of the inflation gap.

The response of the rate rule

$$r_t = r^* + \frac{1}{\zeta_Y \left(\gamma + \frac{1}{\gamma\beta} \right)} \left(\pi_t - \pi^T \right)$$

where

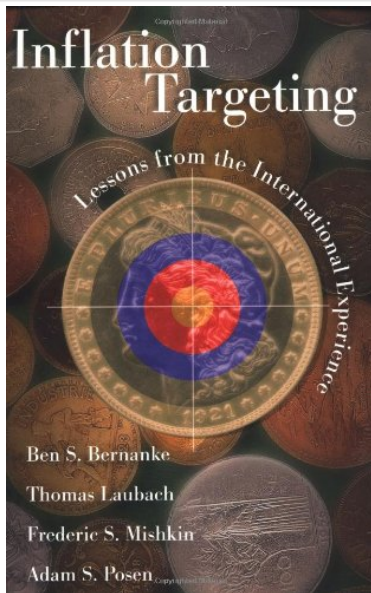
$$r^* = \frac{1}{\zeta_Y} \left(\bar{Y} - Y^P \right)$$

is a function of π^T .

The central bank sets π^T .

The Optimal Rate Rule

Is it inflation targeting?



Users include:

- Czech Republic
- Peru
- Philippines
- Israel
- New Zealand
- United Kingdom
- Canada
- Switzerland
- *United States?*

The Optimal Rate Rule

Is it inflation targeting?

Is the rate rule

$$r_t = r^* + \frac{1}{\zeta_y \left(\gamma + \frac{1}{\gamma\beta} \right)} \left(\pi_t - \pi^T \right)$$

an inflation-targeting rule?

- In a word, no.
- Recall the origin of the optimal rate rule: the minimization of the loss function

$$L = \left(y_t - y^P \right)^2 + \beta \left(\pi_t - \pi^T \right)^2$$

Optimal Shock Response

The decision on how to set the policy rate was settled by considering the central-bank loss function

$$L = \left(y_t - y^P\right)^2 + \beta \left(\pi_t - \pi^T\right)^2$$

which yields the optimal-path relationship

$$\left(y_t - y^P\right) = -\gamma\beta \left(\pi_t - \pi^T\right)$$

and from which we obtained the optimal-rate rule

$$r_t = r^* + \frac{1}{\zeta_y \left(\gamma + \frac{1}{\gamma\beta}\right)} \left(\pi_t - \pi^T\right) .$$

Optimal Shock Response

An analytic (closed-form) solution of this model can be found by substituting the optimal-path relationship

$$(y_t - y^P) = -\gamma\beta (\pi_t - \pi^T)$$

into the Phillips curve

$$\pi_t = \pi_{t-1} + \gamma (y_t - y^P) + \rho_t$$

to generate

$$\pi_t = \pi_{t-1} - \gamma^2\beta (\pi_t - \pi^T) .$$

Optimal Shock Response

Subtracting π^T from both sides of

$$\pi_t = \pi_{t-1} - \gamma^2 \beta (\pi_t - \pi^T) .$$

yields

$$(\pi_t - \pi^T) = (\pi_{t-1} - \pi^T) - \gamma^2 \beta (\pi_t - \pi^T)$$

which will look less cluttered if we write it in terms of the inflation gap $\tilde{\pi}_t = (\pi_t - \pi^T)$ as

$$\tilde{\pi}_t = \tilde{\pi}_{t-1} - \gamma^2 \beta \tilde{\pi}_t .$$

Optimal Shock Response

Bringing $\tilde{\pi}_{t-1}$ in

$$\tilde{\pi}_t = \tilde{\pi}_{t-1} - \gamma^2 \beta \tilde{\pi}_t .$$

to the left-hand side yields

$$\tilde{\pi}_t - \tilde{\pi}_{t-1} = -\gamma^2 \beta \tilde{\pi}_t$$

which, recalling our approximation of the time derivative in our growth-accounting analysis

$$\frac{d\tilde{\pi}_t}{dt} \approx \tilde{\pi}_t - \tilde{\pi}_{t-1} ,$$

can be written as

$$\frac{d\tilde{\pi}_t}{dt} = -\gamma^2 \beta \tilde{\pi}_t .$$

Our equation for the time-evolution of $\tilde{\pi}_t$

$$\frac{d\tilde{\pi}_t}{dt} = -\gamma^2 \beta \tilde{\pi}_t .$$

has the solution

$$\tilde{\pi}_t = \tilde{\pi}_0 e^{-\gamma^2 \beta t}$$

or

$$\left(\pi_t - \pi^T \right) = \left(\pi_0 - \pi^T \right) e^{-\gamma^2 \beta t} .$$

Optimal Shock Response

The half-life of the response to an economic shock along the optimal path

$$\tilde{\pi}_t = \tilde{\pi}_0 e^{-\gamma^2 \beta t}$$

solves

$$\frac{1}{2} = e^{-\gamma^2 \beta t_{1/2}}$$

and is given by

$$t_{1/2} = \frac{\ln(2)}{\gamma^2 \beta} .$$

Optimal Shock Response

An inflation gap that is created in response to an economic shock decays away exponentially. The economic shock manifests itself as a non-zero inflation gap that can come into existence via changes in the variables that define it:

- Target inflation: π^T .
- The Phillips curve: $\pi_t = \pi^e + \gamma (y_t - y^P) + \rho_t$.
- The *IS* curve: $Y_t = \bar{Y} - \zeta_Y r$ because $y_t = \ln(Y_t)$.
- The production function: $Y = AK^\alpha L^{1-\alpha} = K^\alpha (EL)^{1-\alpha}$ because $y^P = \ln(Y^P)$.

Estimating β and γ :

- The value of β is determined by the central bank. We'll assume a β of 1.
- Since $\gamma = |\omega C_{Okun}|$ we can get:
 - ω from the slope of the Phillips curve graphs in lecture 10.
 - C_{Okun} from our analysis of the unemployment and output gaps.

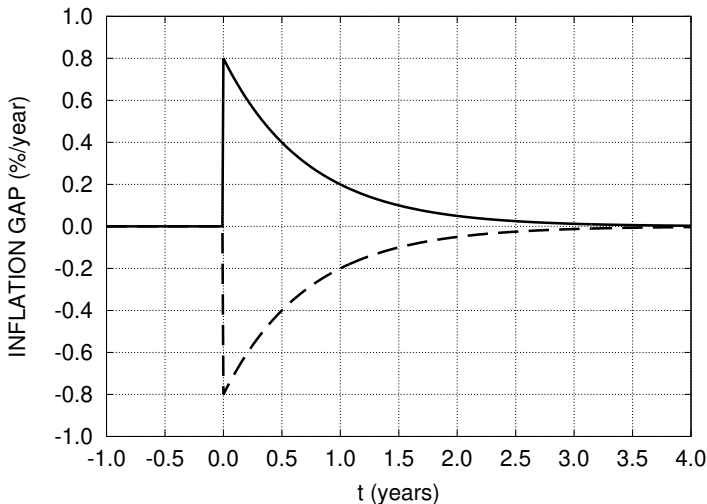
Optimal Shock Response

Estimation results:

Variable	Observation Window	
	1960–1970	1976–1984
ω	-1.49	-2.33
γ	0.89	1.40
$\gamma^2\beta$	0.80	1.95
$t_{1/2}$ (yr)	0.87	0.36
Assumptions: $\beta = 1$ and $C_{Okun} = -0.6$		

Optimal Shock Response

The impulse response for an inflation gap of magnitude ± 0.8 and with $\gamma^2\beta = 1.38$ and $t_{1/2} = 0.5$ years.



Summary:

- The inflation gap undergoes exponential relaxation along the optimal path.
- An inflation gap creates a rate gap at the same time.
- A rate gap creates an output gap in the next time period.
- The subsequent inflation gap is generated by the Phillips curve.